1. Raindrops are falling at an average rate of 20 drops per square inch per minute. What would be a reasonable distribution to use for the number of raindrops hitting a particular region measuring 5 inches2 in t minutes? Why? Using your chosen distribution, compute the probability that the region has no rain drops in a given 3 second time interval. A reasonable choice of distribution is P

The problem describes raindrops falling randomly over a region, which suggests using the **Poisson distribution**. The Poisson distribution is suitable for counting the number of events (raindrops) occurring in a fixed area or interval of time, given that the events occur with a known constant mean rate and independently of each other.

**Details:**

* **Rate of raindrops:** 20 drops per square inch per minute.
* **Region size:** 5 square inches.
* **Time interval:** 3 seconds (which is 360=120\frac{3}{60} = \frac{1}{20}603​=201​ minutes).

**Poisson Distribution Parameters:**

* The mean number of drops λ\lambdaλ for the region and time interval is: λ=rate×area×time=20×5×120=5 drops\lambda = \text{rate} \times \text{area} \times \text{time} = 20 \times 5 \times \frac{1}{20} = 5 \text{ drops}λ=rate×area×time=20×5×201​=5 drops

**Probability Calculation:**

* We need to find the probability of 0 drops in this interval: P(X=0)=λ0e−λ0!=e−λ=e−5P(X = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda} = e^{-5}P(X=0)=0!λ0e−λ​=e−λ=e−5

So, the probability that no raindrops hit the region in the given 3-second time interval is e−5e^{-5}e−5, where e−5e^{-5}e−5 is the answer.

1. Let X be a random day of the week, coded so that Monday is 1, Tuesday is 2, etc. (so X takes values 1, 2,..., 7, with equal probabilities). Let Y be the next day after X (again represented as an integer between 1 and 7). Do X and Y have the same distribution? What is P(X)

Answer :

Let's break down the problem:

* **Random Variable XXX:** Represents a random day of the week, where Monday = 1, Tuesday = 2, ..., Sunday = 7.
* **Random Variable YYY:** Represents the day following XXX, coded in the same way.

**Distribution of XXX:**

XXX is uniformly distributed over the set {1,2,3,4,5,6,7}\{1, 2, 3, 4, 5, 6, 7\}{1,2,3,4,5,6,7}, meaning each day has an equal probability. Therefore:

P(X=x)=17for x∈{1,2,3,4,5,6,7}P(X = x) = \frac{1}{7} \quad \text{for } x \in \{1, 2, 3, 4, 5, 6, 7\}P(X=x)=71​for x∈{1,2,3,4,5,6,7}

**Distribution of YYY:**

YYY is defined as the next day after XXX, so:

* If X=1X = 1X=1 (Monday), Y=2Y = 2Y=2 (Tuesday).
* If X=2X = 2X=2 (Tuesday), Y=3Y = 3Y=3 (Wednesday).
* ...
* If X=7X = 7X=7 (Sunday), Y=1Y = 1Y=1 (Monday).

Therefore, YYY takes values from the same set {1,2,3,4,5,6,7}\{1, 2, 3, 4, 5, 6, 7\}{1,2,3,4,5,6,7} with equal probabilities:

P(Y=y)=17for y∈{1,2,3,4,5,6,7}P(Y = y) = \frac{1}{7} \quad \text{for } y \in \{1, 2, 3, 4, 5, 6, 7\}P(Y=y)=71​for y∈{1,2,3,4,5,6,7}

**Conclusion:**

* **Do XXX and YYY have the same distribution?** Yes, XXX and YYY both have a uniform distribution over the set {1,2,3,4,5,6,7}\{1, 2, 3, 4, 5, 6, 7\}{1,2,3,4,5,6,7}.